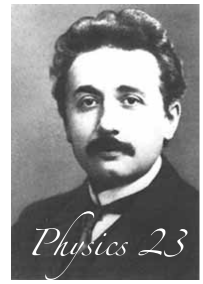


Homework 2 Solution

Tuesday, 11 September 2007

Problem 1 – A-2 Find $\sqrt{401}$ using the binomial approximation. *Hint: First write* $(401)^{1/2} = (400+1)^{1/2} = (400)^{1/2}(1+1/400)^{1/2}$.

Solution: (a) For $|x| \ll 1$, $(1+x)^n \approx 1 + nx$, with the first term neglected being $n(n-1)/2\,x^2$. Taking $x = 1/400$, the binomial approximation gives

$$(401)^{1/2} = (400+1)^{1/2} = (400)^{1/2}(1+1/400)^{1/2} \approx 20\left(1 + \frac{1}{2}\frac{1}{400}\right) = 20 + \frac{1}{40} = 20.025$$

Comparing with a calculator, which gives $\sqrt{401} = 20.02498$ we see that the binomial approximation gives an estimate correct to about 1 part per million. Not bad!

Problem 2 – 4-4 A spacetraveler with 40 years to live wants to see the galactic nucleus at first hand. How fast must the traveler travel? Express your answer in the form $v/c = 1 - \epsilon$, where ϵ is a small number. [We are 8.5 kpc $= 8.5 \times 10^3$ pc from the center of our galaxy. One parsec (pc) $= 3.26\ c$ y.] *Hint:* See Appendix A.

Solution: In the traveler's frame the time interval is τ, where $\tau = 40$ y, so in our frame (i.e., the galaxy's frame), the time interval is *longer*, $\frac{\tau}{\sqrt{1-v^2/c^2}} = \frac{40\text{ y}}{\sqrt{1-v^2/c^2}}$, since to us the traveler's clocks run slow. During this interval the traveler moves a distance $d = vt$, where $d = \left(8.5 \times 10^3 \text{ pc}\right)(3.26\ c\text{ y/pc}) = 2.8 \times 10^4\ c$ y.

Let $v/c = 1 - \epsilon$, so $\sqrt{1 - v^2/c^2} = \sqrt{1-(1-\epsilon)^2} = \sqrt{2\epsilon - \epsilon^2} \approx \sqrt{2e}$, where in the last step we have used the fact that $\epsilon \ll 1$, which implies that $\epsilon^2 \ll 2\epsilon$. Therefore,

$$\frac{40\text{ y}}{\sqrt{2\epsilon}} c(1-\epsilon) = 2.8 \times 10^4 \text{ y}$$

Again, we can neglect the ϵ in $1-\epsilon$, since $\epsilon \ll 1$, so $\frac{40}{2.8\times 10^4} = \sqrt{2\epsilon}$, which can be solved for ϵ to give

$$\boxed{\epsilon = 1.0 \times 10^{-6}}$$

That is, the traveler must move at a speed with respect to the Earth only 1 part in a million slower than the speed of light in order to reach our galactic nucleus in 40 years from the traveler's point of view.

Problem 3 – 4-7 A spaceship leaves the solar system at speed $v = \frac{3}{5}c$, headed for the star Sirius, 10 c y away. Assume that the Sun and Sirius are mutually at rest, and that their clocks have been previously synchronized, both reading zero when the spaceship leaves. (a) What will the clock on Sirius read when the ship arrives? (b) What will the clock on the ship read when it arrives, assuming it began at zero? (c) In the ship's frame of reference, what will the Sun's clock read when the ship arrives?

Solution: (a) The ship must travel $D = 10\ c$ y at speed $3/5\,c$, so it takes $\Delta t = D/c = 16\frac{2}{3}$ y.

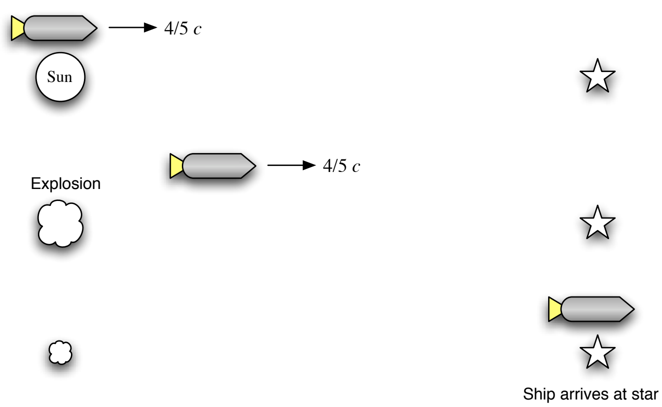
(b) The rocket's clock began at 0 and ran slow for a period of $16\,\frac{2}{3}$ y on the Earth's clock. Therefore, it will read $t_{r,A} = \sqrt{1 - v^2/c^2}\,\left(16\frac{2}{3}\text{ y}\right) = 13\frac{1}{3}$ y.

(c) From the ship's perspective, the Sun left the ship when both the Sun's and the ship's clock read 0. The Sun's clock is a moving clock and ticks slow by the factor $\sqrt{1 - v^2/c^2}$. Therefore, the Sun's clock reads

$$t_{s,A} = t_{r,A}\sqrt{1 - v^2/c^2} = \left(\frac{40}{3}\text{ y}\right)(4/5) = 10\frac{2}{3}\text{ y}$$

Problem 4 – 4-10 The Sun and another star are 60 light-years apart and are at rest relative to one another. At time $t = 0$ (on both the Sun clock and the spaceship clock) a spaceship leaves the Sun at velocity $v = \frac{4}{5}c$ headed for the star. Just as the ship arrives, a light-signal from the Sun indicates that the Sun has exploded. (a) Draw a set of three pictures in the rest-frame of the Sun and star for the three important events in the story, *i.e.,* at the time of the ship departure, the Sun explosion, and the ship arrival. Each picture should show the Sun, the star, and the ship. Then answer the following questions, all from the point of view of observers at rest in the Sun-star frame. (b) When the ship arrives, what does the star clock read? (c) When the ship arrives, what does the ship clock read? (d) When the ship arrives, what does the Sun clock read? (e) When the Sun explodes, what do clocks on the Sun, the star, and the ship read?

Solution:



(b) The ship takes $(60\ c\text{ y})\,/\left(\frac{4}{5}\text{ c}\right) = 75$ y to arrive at the star, according to clocks in the Sun-star system.
(c) The ship's clock runs slow by a factor of $\frac{3}{5}$ for 75 y, starting from 0. Therefore, it reads $\frac{3}{5}(75\text{ y}) = 45$ y when it arrives at the star.
(d) The Sun's clock is synchronized with the star's in this frame. Hence, it reads 75 y, just as the star's does.
(e) By the definition of the light-year, it takes a light signal 60 y to travel the 60 c y from Sun to star. Since it arrives 75 y after the ship left, the explosion must have taken place 15 y after departure, according to clocks on the Sun and star. This ship's clock, however, runs slow by the factor of $\frac{3}{5}$, so it reads only $\frac{3}{5}(15\text{ y}) = 9$ y.